SERIES WORKSHEET 1

Problem 1. Decide whether the series converge absolutely, conditionally, or if they diverge.

(1) $\sum_{n=1}^{\infty} \frac{(2n)!}{(3n)!}$, (2) $\sum_{n=1}^{\infty} \frac{n^{50}50^n}{n!}$, (3) $\sum_{n=1}^{\infty} \frac{n^2}{(-3)^n}$, (4) $\sum_{n=1}^{\infty} \frac{10^n}{(n+3)4^{2n-1}}$,

(5) $\sum_{n=1}^{\infty} \frac{5^n}{3^n+4^n}$, (6) $\sum_{n=1}^{\infty} \frac{n!}{2^{n^2}}$, (7) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^{\frac{5}{3}}}$, (8) $\sum_{n=1}^{\infty} \left(1-\frac{1}{n}\right)^{n^2}$,

(9) $\sum_{n=1}^{\infty} \frac{1}{\sqrt[n]{n!}}$, (10) $\sum_{n=1}^{\infty} e^{-\sqrt{n}}$, (11) $\sum_{n=1}^{\infty} \left(\frac{n^2}{e^n} - \frac{n^2}{1+n^3}\right)$, (12) $\sum_{n=2}^{\infty} \frac{\cos(\pi n)}{\ln n}$,

(13) $\sum_{n=1}^{\infty} \frac{1}{n^{1+\sin\frac{1}{n}}}$, (14) $\sum_{n=1}^{\infty} \sin(e^{-n})$, (15) $\sum_{n=1}^{\infty} \frac{1}{n^{1+\frac{1}{\ln(\ln n)}}}$, (16) $\sum_{n=1}^{\infty} e^{-(\ln n)^2}$,

Problem 2. Let $(a_n)_n, (b_n)_n$ be sequences of real numbers. Decide with justification (proof or counterexample) whether the statement is true:

(a) ? If $\sum_{n=1}^{\infty} |a_n|$ converges and $(b_n)_n$ is bounded, then $\sum_{n=1}^{\infty} a_n b_n$ converges ?

(b) ? If $\sum_{n=1}^{\infty} a_n$, $\sum_{n=1}^{\infty} b_n$ both diverge, then so does $\sum_{n=1}^{\infty} a_n + b_n$?

(c) ? If $\sum_{n=1}^{\infty} a_n$ converges, then the sequence $(na_n)_n$ is bounded ?

(d) ? If $a_n \geq 0$ for all n and $\sum_{n=1}^{\infty} a_n$ converges, then so does $\sum_{n=1}^{\infty} a_n^2$?

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